Quiz 7, MATH 240, Fall 2023

Write your name clearly.

Name:

UID:

(1) (12 points) Let $\mathcal{B} = \left\{ \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ be a basis for $\mathbb{R}^2$. (You can assume it's a basis.)

(a) Find the change-of-coordinates matrix $P_{\mathcal{B}}$ to change from $\mathcal{B}$-coordinates to the standard basis.

(b) Find $(P_{\mathcal{B}})^{-1}$.

(c) Use (b) to find the coordinate vector $\left[\begin{pmatrix} 1 \\ 4 \end{pmatrix} \right]_{\mathcal{B}}$.

(a) $P_{\mathcal{B}} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

(b) $P_{\mathcal{B}}^{-1} = \frac{1}{6-5} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

(c) $[(1,4)]_{\mathcal{B}} = P_{\mathcal{B}}^{-1}(1,4)$

$= \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 7 \end{pmatrix}$.

(2) (8 points) The RREF of matrix $A = \begin{pmatrix} 1 & -4 & -11 & 6 \\ -3 & -4 & 1 & -3 \\ 7 & 10 & -1 & -2 \\ 4 & 6 & 0 & -5 \end{pmatrix}$ is $\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

Write down a **basis** for $\mathrm{Col}(A)$ and $\mathrm{Row}(A)$. (No calculation required!)

Col(A) has basis $\{(1,-3,7,4), (-4,-4,10,6), (6,-3,-2,-5)\}$.

Row(A) has basis $\{(1\ 0\ -3\ 0), (0\ 1\ 2\ 0), (0\ 0\ 0\ 1)\}$.